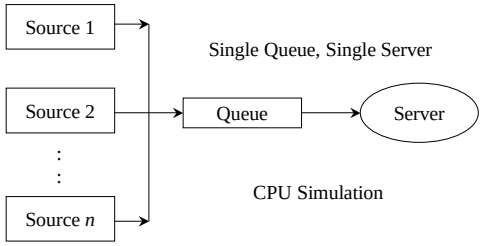
Discrete Event Systems

The Basics

Discrete Event Systems

(Queueing Systems)



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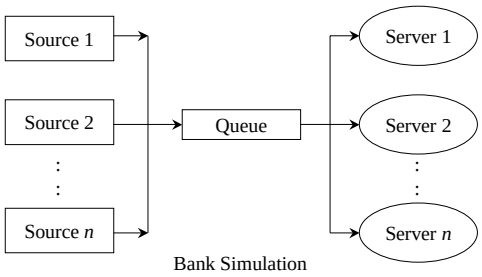
Discrete Event Systems

Single Queue, Multi-Server



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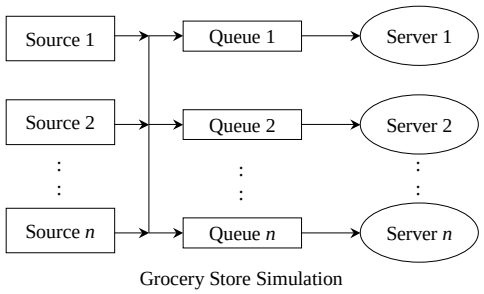
Discrete Event Systems

Multi-Queue, Multi-Server



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Queueing Systems

(Parameters)

- Arrival Rate
  - Inter-arrival time
- Arrival Distribution
- Average (Mean) Service Time
- Service Distribution
- Travel Time (optional)
  - Transition time

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Queueing Systems

(Outputs)

- Average Queueing Time
  - Average Wait Time
- Average Queue Length
  - Congestion
- System Throughput
  - Avg. # of completions per unit time
- Server Utilization
  - %'age of time server is busy

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Probability Basics

- (Pseudo) Random Number Generators
- Probability Density Functions (PDFs)
  - Probability Distribution Functions
- Cumulative Density Functions (CDFs)
  - Cumulative Distribution Functions
- Random Variates

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(Pseudo) Random Number Generators

- Generate uniformly distributed numbers in the range (0.0 ~ 1.0)
- The probability of getting any *one* number should be the same for *all* numbers in the range (i.e. the Probability Density Fn. (PDF) is a straight line)
- The generator should be fast and not require large amounts of memory
- The generator should be able to regenerate a series of random numbers (from a *seed*) for debugging purposes.

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Dist'n of Telephone Call Durations

Call Len.	# of Calls	Freq.	Prob.	Cum.
0	0	-	-	-
10	1	0.001	0.001	0.001
20	1	0.001	0.001	0.002
30	1	0.001	0.001	0.003
40	2	0.002	0.002	0.005
50	8	0.008	0.008	0.013
60	28	0.028	0.028	0.041
70	65	0.065	0.065	0.106
80	121	0.121	0.121	0.227
90	175	0.175	0.175	0.402
100	197	0.197	0.197	0.599
110	175	0.175	0.175	0.774
120	121	0.121	0.121	0.895
130	65	0.065	0.065	0.960
140	28	0.028	0.028	0.988
150	9	0.009	0.009	0.997
160	2	0.002	0.002	0.999
170	1	0.001	0.001	1.000

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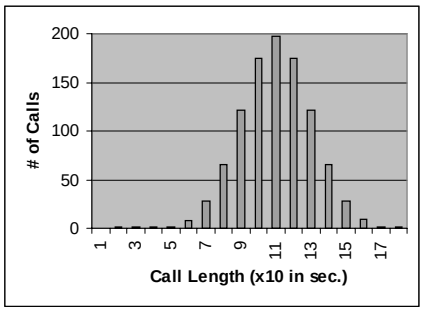
Probability Density Function



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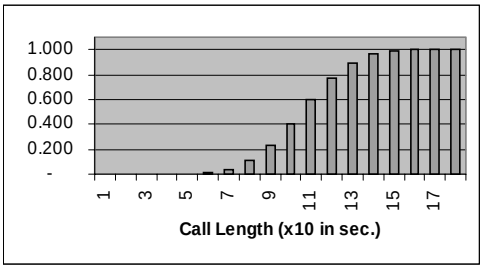
Cumulative Density Function



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Relationship between PDF & CDF

$$cdf(x) = \int_{0}^{x} pdf(x)\,dx$$

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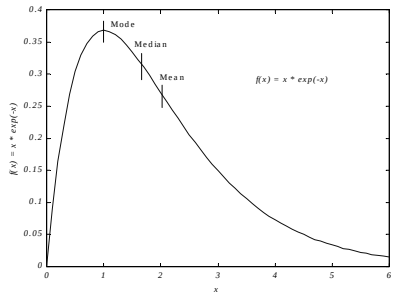
A Probability Density Function



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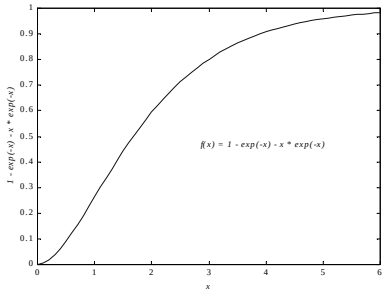
A Cumulative Density Function



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Problem Formulation

- Identify *decision* and *uncontrolled* variables
- Specify *constraints* on the decision variables
- Define *measures* of system performance and an objective function
- Develop a *preliminary model* structure to interrelate the system variables and the performance measures

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Grocery Store Example

- A Supermarket has several checkout lines staffed by Cashiers (Checkers) & Baggers
- Customers arrive and choose one of the checkout lines (Express Lane for < 10 items)
  - Enter the system when a lane is chosen
  - Leaves the system when the bill is paid
- Operations included in the system:
  - Unload cart, Tally items, Bag items, Payment

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Variables and Constraints

- Exogenous Variables
  - Variables *outside* the system
  - *Independent* variables
  - *Input* variables to the system
  - Can be *controllable* or *uncontrollable*
    - Controllable variables are called *decision variables*
    - Uncontrollable variables are called *parameters*
- List the exogenous variables in our system!

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Exogenous Variables

Uncontrollable

- Number of checkout lanes in the store
- Number of lanes with Cashiers only
- Number of lanes with Cashiers & Baggers
- Checkout equipment used (speed)
- Number and size of Express Lane
- Arrival pattern of Customers
- Salaries of Cashiers and Baggers

Controllable

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Variables and Constraints

- Endogenous Variables
  - Variables *inside* the system
  - *Dependent* variables
    - Dependent on Exogenous vars & Model structure
  - *Output* variables from the system
  - These system variables determine the detail of our model (by their quantity and range)
- List the endogenous variables in our system!

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Endogenous Variables

- Number of Customers waiting in line
- Customer waiting time
- Cashier idle time
- Bagger idle time
- Customer's total checkout time
  - (Queueing and Service Time)

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Objective Function

- The function to be maximized or minimized
  - Maximize throughput
  - Minimize average wait time
  - Maximize profit
  - Minimize salary costs
- Several of these are conflicting
  - Improving one will adversely affect another
- Combining several goals into one objective

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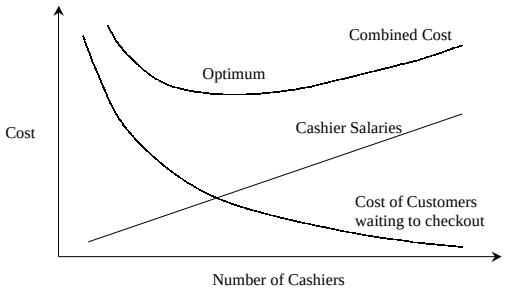
Cost Relationships in our system



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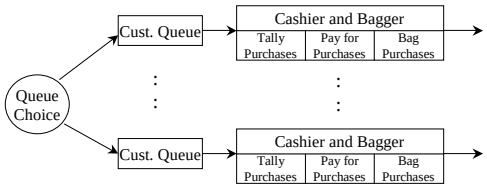
Model Development

- Physical Flow Approach
  - Flow Diagram:



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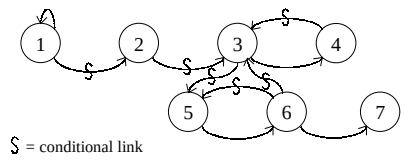
Event Graphs

- State Transition Graphs:

1. Customer arrives
2. Customer Selects Cashier
3. Cashier begins checkout
4. Cashier finishes checkout
5. Bagger begins packing
6. Bagger finishes packing
7. Customer leaves store



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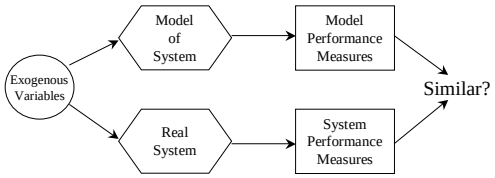
Model Verification & Validation

- Model debugging
  - But how does one know when we're done?
- Model Validation:



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Output Analysis

- Boundary Conditions
  - System start-up (Beginning of day)
  - System shutdown (End of day)
- Peak Periods
- Steady State Analysis
- Statistical Methods
  - ANOVA (ANalysis Of VAriance)
  - RSM (Response Surface Method)

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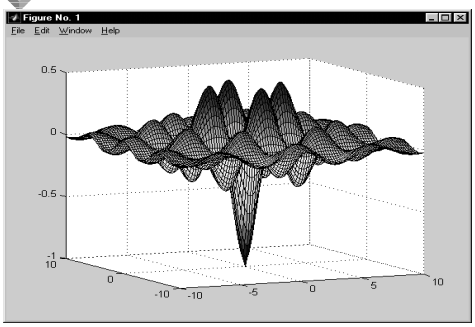
Response Surface Method



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