Developing Simulation Models

(Chapter 3)

Buffon Needle Experiment

- A simple experiment to calculate π:
  - Uses random trials
  - By dropping needles on the floor
    - Imagine an old schoolhouse with plank floors
    - Each plank is a distance *d* in width (i.e. the distance between the lines on the floor is *d*)
    - The needle has length *l* (obviously, $l < d$)
    - Let *NI* = # intersecting needles (they touch a line)
    - Let *NT* = total # of trials
    - Let $p = (NI)/(NT)$ = probability of intersecting

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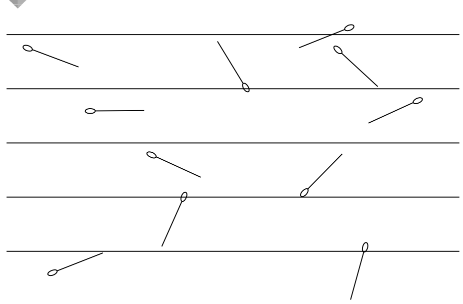
Buffon Needle Experiment



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Monte Carlo Simulation

- The Buffon Needle experiment represents a *Monte Carlo* simulation
  - Monte Carlo methods are probabilistic methods that produce a result that may contain some error
  - Random numbers and random sampling will be used in our experiments
  - We will need to simulate the dropping of needles on the floor in a computer program

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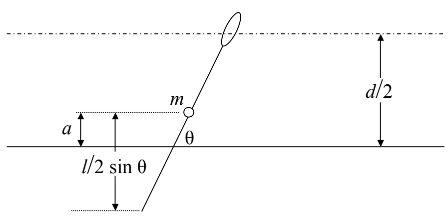
Simulation of Buffon Needle Exp.

- Every pin can be described by its midpoint m and its angle θ to the parallel lines



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Simulation of Buffon Needle Exp.

Each needle is described by a and θ

- If $a < l/2 \sin\theta$ then the pin touches the line
- To obtain random samples of a and θ we'll use pseudo-random numbers where $0 \le r \le 1$

$$a = (d/2)r$$
$$\theta = \pi r$$

The ranges for a and θ are:

$$0 \le a \le d/2$$
$$0 \le \theta \le \pi$$

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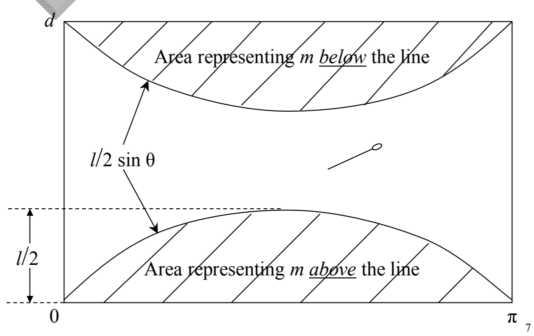
Solution Space for p = NI/NT



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Calculating p

- *p* = probability that a needle touches a line
- *p* = NI /NT = (Number Intersecting line) / (Total number of trials)
- *p* = the ratio of the shaded region of the rectangle to the area of the entire rectangle (on the previous slide)

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Statistics of Buffon's Needle

$$p = 2\int_0^{\pi} \frac{(l/2)\sin\theta \, d\theta}{\pi d}$$

$$p = \frac{2l}{\pi d}$$

$$\pi = \frac{2l}{p\,d}$$

From Fig 3.4 p. 43
Simulation: Hoover & Perry

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Experiment #1

- Conduct the Buffon's Needle experiment
  - Do the physical experiment
    - Physically drop "needles"
    - Compute your estimate of π
  - Write a simulation program outlined here
  - Compare the results
  - Notice any similarity?

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Back to the Grocery Store!

- In a manner similar to Buffon's Needle, we can conduct a Grocery Store simulation
- First, let's produce an event representation that will account for the salient events that occur within our Grocery Store system
  - We'll use an Event Graph
    - Introduced in Chapter 2 of Hoover & Perry
  - Let's track the number of customers in the store

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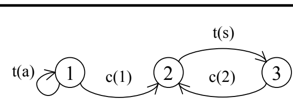
Event Graph



State Variable

n = number of customers in the system

Events

1 customer arrives (enter queue)

2 customer leaves queue/service begins

3 service completed

Conditions

c(1): n = 0

c(2): n > 0

Delays

t(a) = time until next arrival

t(s) = time to service a customer

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Time Table

Hoover & Perry, page 48

Arrival #	Time of Arrival	t(a)	t(s)
1	4	6	3
2	10	3	5
3	13	6	6
4	19	1	4
5	20	9	7
6	29	2	3
7	31	3	6
8	34	9	7
9	43	4	2
10	47	4	6
11	51	7	3
12	58	5	8

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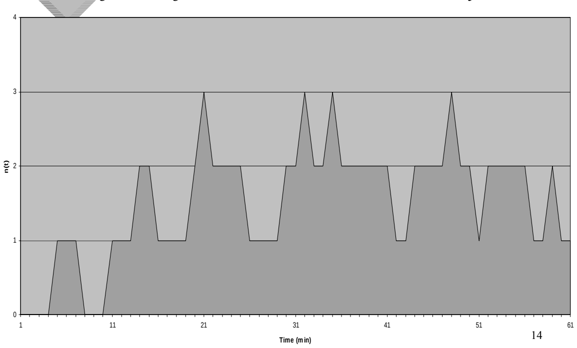
Profile of Customers in the System



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Analytical Analysis

- ❖The area under the curve represents the # of minutes all customers spent in the system
- ❖If we divide total customer time by the # of customers who entered the store we get:
  - ❖Estimate of the mean time spent per customer
- ❖If we divide total customer time by the length of the simulation we get:
  - ❖Estimate of the mean # of customers in system

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More Formally

$n(t) = \#\ of\ customers\ in\ system\ at\ time\ t$

$k = \#\ of\ arrivals$

$T = length\ of\ the\ simulation$

$\overline{n} = mean\ \#\ of\ customers\ in\ system$

$\overline{w} = mean\ time\ a\ customer\ is\ in\ system$

$$\overline{n} = \frac{1}{T}\int_0^T n(t)\,dt$$

$$\overline{w} = \frac{1}{k}\int_0^T n(t)\,dt$$

A complete development of the Grocery Store simulation pgm is described in detail in H & P pp. 50 – 52. Fortran code is even provided in the Appendix!

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What Can We Deduce?

- If the demand for service does not exceed the capacity of the system, the following holds:

$$\lim_{k\to\infty}\left[p(|\overline{w} - E(w)|) < \varepsilon\right] = 1.0$$

$$\lim_{T\to\infty}\left[p(|\overline{n} - E(n)|) < \varepsilon\right] = 1.0$$

- In other words, by increasing the length of the simulation we become more certain that *w* and *n* will be within ε units of their expected values

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Simulation of Same System

- Let's simulate the behavior of our Grocery Store with regard to customers arriving, checking out, and leaving the store
- An Events List is on page 48 of H & P
  - It completely accounts for every significant event that occurs during the simulation
    - Notice that multiple events can happen at the same instant of time

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Simulation Variables

N = Number of customers currently in system

T = Current time

T_{max} = Duration of the simulation

T_s = Service time

T_a = Arrival time of next customer

K = Total # of arrivals during simulation

TN = Time integral of N

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Single Queue, Single Server

❖A single register grocery store simulation

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