Summary of “On the Modeling and Analysis of Computer Networks” by Leonard Kleinrock

Kenneth J. Christensen

Department of Computer Science and Engineering
University of South Florida
Tampa, Florida 33620
christen@csee.usf.edu

College of Engineering
University of South Florida

Reference

- L. Kleinrock, "On the Modeling and Analysis of Computer Networks," *Proceedings of the IEEE*, Vol. 81, No. 8, August 1993.

- Some facts on Leonard Kleinrock
  - Homepage is *http://millennium.cs.ucla.edu/lk.html*
  - Ph.D. from MIT in 1963
  - "Father of Modern Data Networking" (invented packet switching)
  - Published the classic text on queueing theory
  - Laid out key functional specifications for the ARPANET
  - Produced 39 Ph.D.'s - all are leaders in the field
  - Numerous professional awards
  - Currently is chair of UCLA Computer Science department

Overview of paper

- An overview paper of,
  - "… the landscape of analytic models for computer network performance evaluation."

- Paper organization
  - Section I - Introduction and Apology
  - Section II - The Early Network Analysis Models
  - Section III - Design Issues
  - Section IV - Control Issues (routing and flow control)
  - Section V - Gigabit Networks
  - Section VI - Conclusions
  - References - 22 references

Section 1 - Introduction and Apology

- Goal of performance evaluation…
  - To be able to predict how a network will perform

- Four ways (in order of ugliness = cost)
  1) Mathematical analysis yielding explicit expressions
  2) Mathematical analysis yielding numerical procedure
  3) Simulation study
  4) Experimental study (build the actual system)

➡ Focus of this paper is on the first way

Section 1 - Introduction and Apology

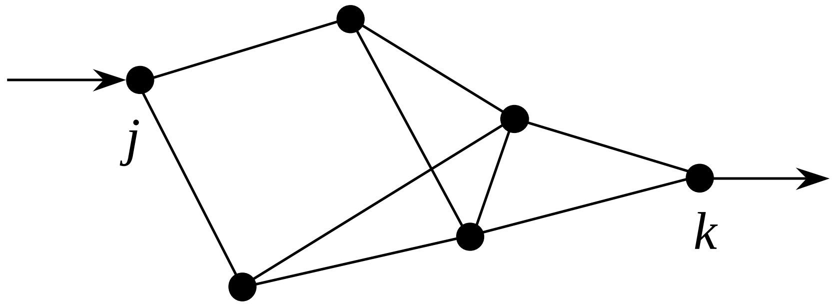
- All models are an approximation
  - Exact solution is not necessary

- Sometimes makes sense to use an even more approximate model if solution is more tractable (than that of a less approximate model)

Section 2 - The Early Network Analysis Models

- Goal: Expression for delay in a network

- Network model...



M nodes
N links
Origin *k*
Destination *j*

» Message arrivals are Poisson,
  · Rate γ_{jk} messages per second
  · Mean message size is $1/\mu$ bits
  · Channel capacity is C_i bits/sec
» Each node is a queueing system

Section 2 - The Early Network Analysis Models

- Total external traffic (γ), total internal traffic (λ), and mean number of hops per message ($\bar{n}$)

$$\gamma = \sum_{j,k=1}^{N} \gamma_{jk}, \quad \lambda = \sum_{i=1}^{M} \lambda_i \text{ , and } \quad \bar{n} = \frac{\lambda}{\gamma}$$

- We get the last expression as follows...

$$\bar{n} = \sum_{j,k} \frac{\gamma_{jk}}{\gamma} \cdot n_{jk}$$

Substituting...

$$\lambda = \sum_{i} \lambda_i = \sum_{j,k} \gamma_{jk} \cdot n_{jk}$$

Section 2 - The Early Network Analysis Models

- And, thus the mean response time of the network is,

$$T = \sum_{i=1}^{M} \frac{\lambda_i}{\gamma} T_i$$

Need an explicit expression for T_i

Section 2 - The Early Network Analysis Models

- There is no accurate and tractable expression for T_i
  - There is dependence among successive service times
  - Would like to “wish away” this dependence

- The famous Kleinrock *Independence Assumption*
  - Assume that a message length is chosen independently from the exponential distribution each time it enters a switching node in the computer network.

➡ Now each node is an M/M/1 queue!

Section 2 - The Early Network Analysis Models

- Quick review of M/M/1 “formulas”

$\lambda =$ *message arrival rate and* $\mu =$ *message service rate*

$$\rho = \text{utilization} = \frac{\lambda}{\mu}$$

$$T = \text{system delay} = \frac{1}{\mu - \lambda}$$

$$W = \text{queueing delay} = \frac{\rho}{\mu - \lambda}$$

$$N = \text{mean number customers in system} = \frac{\rho}{1-\rho}$$

You should be able to derive all of these!!!

Section 2 - The Early Network Analysis Models

- Then,

$$T_i = \frac{1}{\mu \cdot C_i - \lambda_i}$$

$$T = \sum_{i=1}^{M} \frac{\lambda_i}{\gamma} \left(\frac{1}{\mu \cdot C_i - \lambda_i} \right)$$

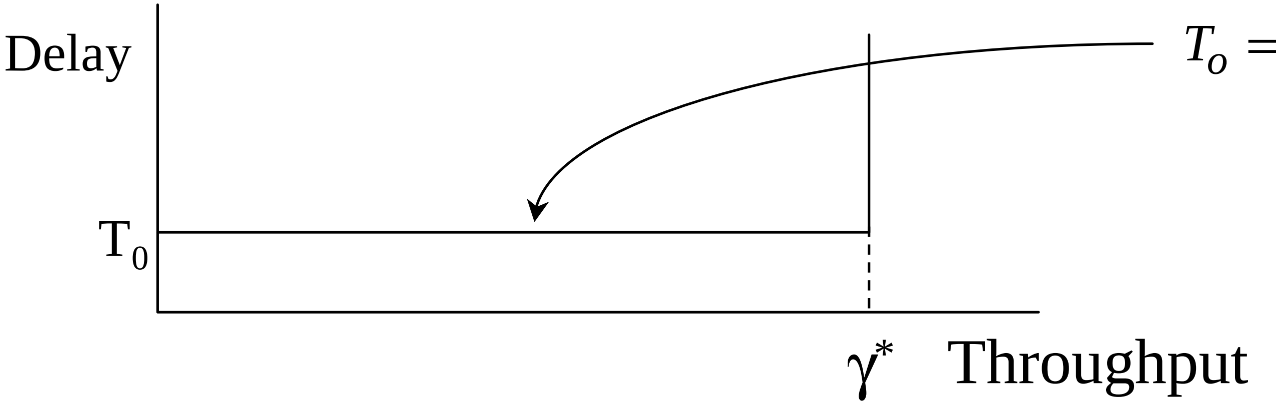
Section 2 - The Early Network Analysis Models

- We can add more accuracy
  - K = nodal processing time
  - P_i = propagation delay for channel i
  - $1/\mu$ = mean length of data packets only in bits
  - $1/\mu'$ = mean length of all packets (data and control)

$$\boxed{T = K + \sum_{i=1}^{M} \frac{\lambda_i}{\gamma}\left(\underbrace{\frac{\lambda_i / \mu'\cdot C_i}{\mu'\cdot C_i - \lambda_i}}_{W_i} + \underbrace{\frac{1}{\mu \cdot C_i}}_{\text{Service time}} + P_i + K \right)}$$

Section 2 - The Early Network Analysis Models

- Thresholding behavior
  - – Assume that C_i and λ_i are fairly homogeneous
  - – No individual term will dominate the summation
  - – Until… the flow in one channel approaches channel capacity



$$T_o = \sum_{i=1}^{M} \frac{\lambda_i}{\gamma} \frac{1}{\mu \cdot C_i}$$

γ^* = smallest γ at which some critical channel is saturated

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Section 3 - Design Issues

- Difficulties include,
  - Cost and link capacity relationship not linear
  - Tariffs are “weird and illogical”
  - Performance and load relationship is not linear
  - Number of possible topologies is virtually infinite
  - Data are missing and/or inaccurate

Section 3 - Design Issues

- “Design” means selecting design parameters to...
  - Optimize an objective function
  - While meeting all system constraints

⇒ Formulate as a classic Operations Research minimization problem

Section 3 - Design Issues

- List of parameters…
  1) Delay
  2) Topology
  3) Channel capacity assignment
  4) Routing procedure
  5) Budget in dollars
  6) Traffic matrix to be satisfied

Section 3 - Design Issues

- Problem #1
  - Minimize (1) by selecting (2), (3), and (4) such that (5) and (6) are satisfied

- Problem #2
  - Minimize (5) by selecting (2), (3), and (4) such that (1) and (6) are satisfied ($T < T_{max}$)

- Problem #3 (Flow Assignment problem)
  - Given (2) and (3) minimize (1) by selecting (4) such that (6) is satisfied

Section 3 - Design Issues

- Problem #4 (Capacity Assignment problem)
  - Given (2) and (4) minimize (1) by selecting (3) such that (5) and (6) are satisfied

- Problem #5 (Capacity and Flow Assignment problem)
  - Given (2) minimize (1) by selecting (3) and (4) such that (5) and (6) are satisfied

Problem #5 has many local minima… is thus a challenging problem to solve

Section 3 - Design Issues

- Economies of scale
  - As N increases, the decrease in bandwidth for each additional node decreases relatively less

Section 4 - Control Issues

- Routing control
  - "… a decision rule which determines where next to send a packet as it travels from switching node to switching node."

- Method can be distributed or centralized

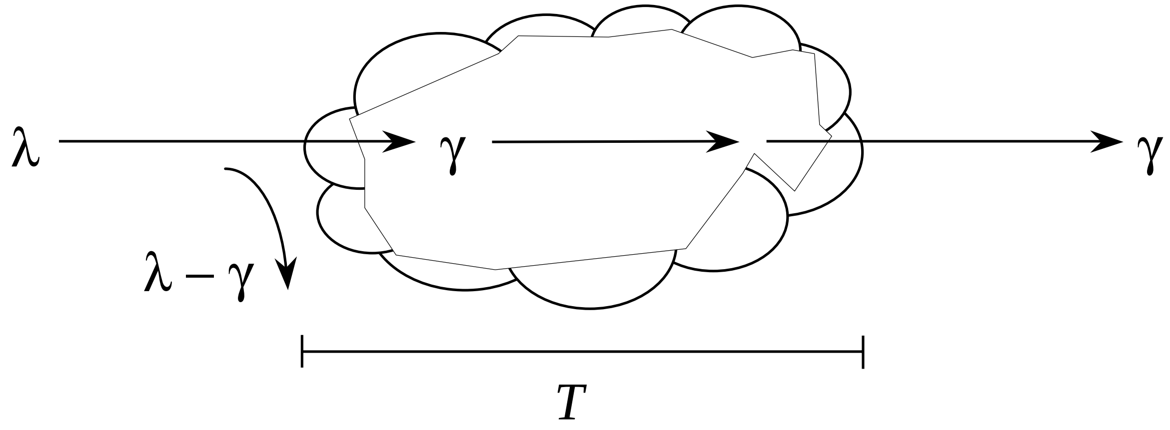
- Original ARPANET distributed routing procedure
  - Node table contains estimates of delay to all other nodes
  - Receive neighbor's table, estimate delay to neighbor, add this delay to entries in table
  - Then, select and use minimum estimate across all neighbors for a given destination

Section 4 - Control Issues

- Flow control
  - Prevent host from overflowing a terminal and thus also wasting network bandwidth
  - Prevent terminal from excessively interrupting a host
- We wish to…
  - Throttle input flows at network boundary
- Window flow control
  - Can transmit no more than $W - 1$ message past the first unacknowledged message for a window size of W

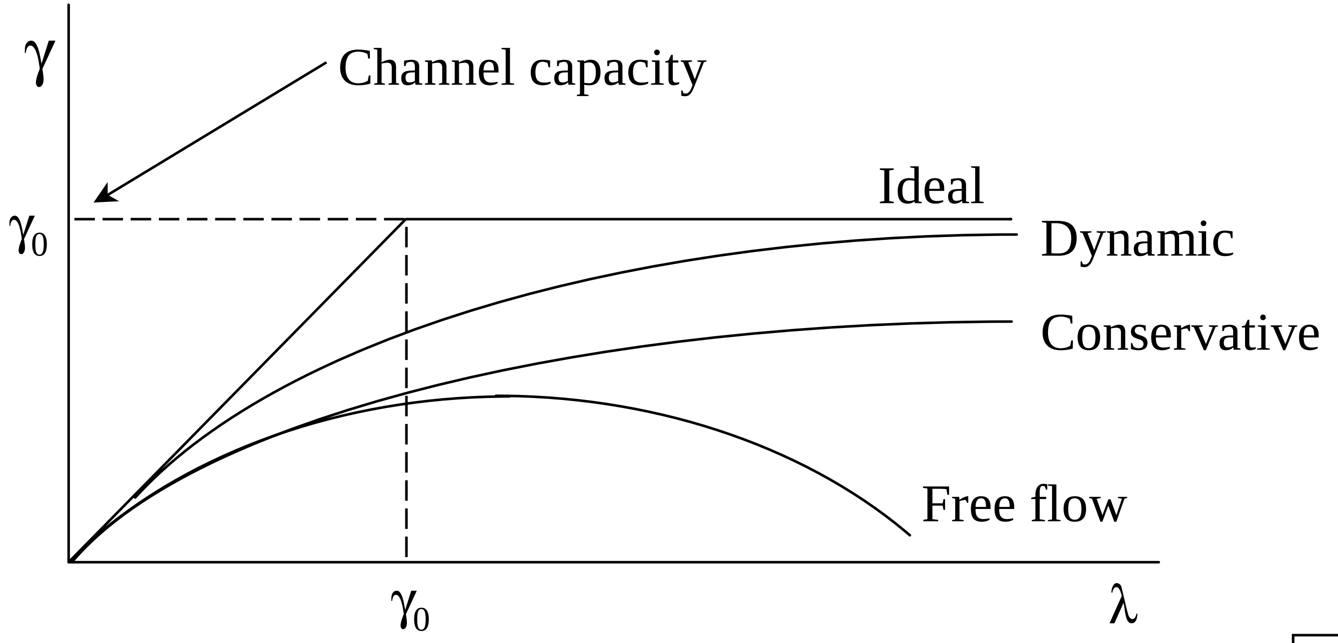
Section 4 - Control Issues

- Three competing performance measures
  - – Throughput
  - – Response time
  - – Loss (or blocking)



Section 4 - Control Issues

- Flow control function
  - Throughput is $\gamma(\lambda)$
  - Mean response time is $T(\gamma(\lambda)) = T(\gamma)$



Section 4 - Control Issues

- We want high throughput and low delay
  - Define network "power", *P*, as,

$$P = \frac{\gamma(\lambda)}{T(\gamma)}$$

- We wish to find value of λ which maximizes *P*

$$\frac{dP}{d\lambda} = \frac{T(\gamma)\cdot d\gamma(\lambda)/d\lambda - \gamma(\lambda)\cdot dT(\gamma)/d\lambda}{[T(\gamma)]^2} = 0$$

which is,

$$T(\gamma)\cdot\frac{d\gamma(\lambda)}{d\lambda} = \gamma(\lambda)\cdot\frac{dT(\gamma)}{d\lambda}$$

Section 4 - Control Issues

- Working this out for *k* hops of M/M/1…

$$T(\gamma) = \frac{k}{\gamma_0 - \gamma(\lambda)}$$ where γ_0 is the channel capacity.

We have that,

$$\frac{dT(\gamma)}{d\lambda} = \frac{k \cdot d\gamma(\lambda)/d\lambda}{[\gamma_0 - \gamma(\lambda)]^2}$$

And thus,

$$\left[\frac{k}{\gamma_0 - \gamma(\lambda)}\right] \cdot \frac{d\gamma(\lambda)}{d\lambda} = \gamma(\lambda) \cdot \left[\frac{k \cdot d\gamma(\lambda)/d\lambda}{[\gamma_0 - \gamma(\lambda)]^2}\right]$$

Section 4 - Control Issues

- Working this out for *k* hops of M/M/1… (continued)
  - Cancel $d\gamma(\lambda)/d\lambda$
  - Multiply through by $(\gamma_o - \gamma(\lambda))/k$
  - And we get,

$$= \frac{\gamma(\lambda)}{\gamma_0 - \gamma(\gamma)}$$

  - And then,

$$\gamma(\lambda) = \frac{\gamma_0}{2}$$

➡ Optimum throughput is 1/2 of channel capacity

Section 4 - Control Issues

- Working this out for *k* hops of M/M/1… (continued)
  - Thus for maximized power for a single M/M/1,

$$\rho = 0.50$$

  - For which $N = 1$ (average number of customers)

For an *N* hop path, there should be *N* messages "in flight" in the network (or $W = N$)

Section 5 - Gigabit Networks

- Are gigabit speed networks “different” from existing networks?

- Define the “*a*” ratio as

$$a = \frac{5 \cdot L \cdot C}{b}$$

- where,
  - C = capacity of network in Mbps
  - b = number of bits in a message
  - L = length of network in miles
  - 5 = number of microseconds for light to travel 1 mile

Section 5 - Gigabit Networks

- Table of *a* values

Network type	**Capacity (Mbps)**	**Packet len (bits)**	**Prop delay (μsec)**	**“*a*” value**
LAN	10.00	1000	5	0.05
WAN	0.05	1000	20,000	1.00
Satellite link	0.05	1000	250,000	12.50
Fiber across USA	1000.00	1000	15,000	15,000.00

A range of six orders of magnitude

Section 5 - Gigabit Networks

- Consider an example, sending a 1 Megabit file across the USA
  - At 64-kbps, network is about 1000 bits or 0.0001 of file
  - At 1.544-Mbps, network = 24,000 bits or 0.025 of file
  - At 1.2-Gbps, network is about 20,000,000 bits

At 1.2-Gbps the entire file fits on the network 20 times… implications to closed-loop flow control

Section 5 - Gigabit Networks

- Concept of l*atency-limited* versus *bandwidth-limited*
  - Define boundary as,

propagation delay = queueing plus transmission delay

- For M/M/1 with mean *b* bits message length

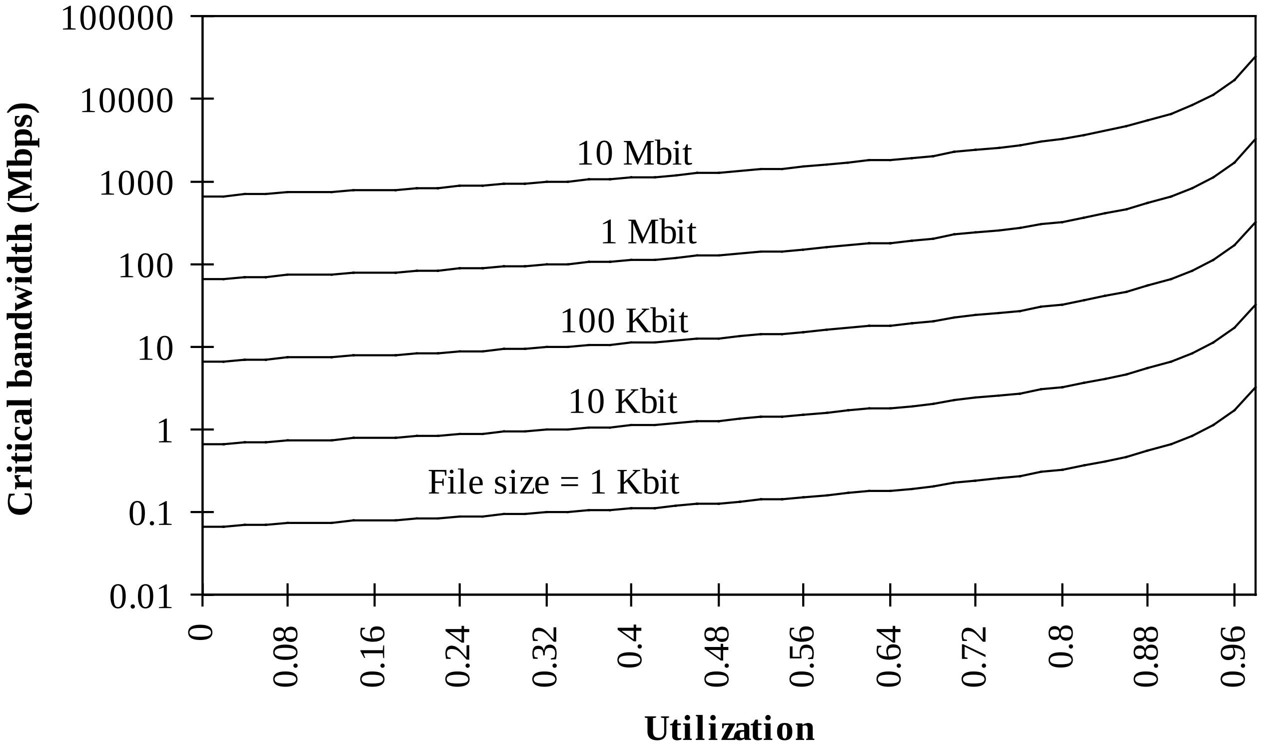
$$T = \frac{b}{1-\rho} + \tau \quad \text{where } \tau \text{ is the propagation delay}$$

- And thus *critical bandwidth* is,

$$C_{critical} = \frac{b}{(1-\rho)\cdot\tau}$$

Section 5 - Gigabit Networks

- Plot of $C_{critical}$ for cross-country link (τ = 15,000 μsec)



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Section 5 - Gigabit Networks

- Maximum attainable efficiency
  - From statistical multiplexing
  - That is,

$$Allocated\ bandwidth > \sum peak\ bandwidth\ per\ connection$$

- Possible due to Law of Large Numbers if many small, homogeneous bursty sources

Section 5 - Gigabit Networks

- Recent work (not discussed) shows that Law of Large Numbers does not work
  - Problems with “self-similarity” of traffic
  - See, for example, recent works by Willinger, Leland, Taqqu, et al.

Section 6 - Conclusion

- Have completed a "tour" of analytic modeling of computer networks

- With Gigabit networks,
  - "The switch as become the new economic and performance bottleneck of the system"
  - "New protocols and architectures require modeling, analysis and design"