All men, except butchers, like all vegetarians.

$$\forall X\ \forall Y\ male(X) \wedge \neg butcher(X) \wedge vegetarian(Y) \Rightarrow likes(X,Y)$$

All male butchers do not like vegetarians.

$$\forall X\ \forall Y\ male(X) \wedge butcher(X) \wedge vegetarian(Y) \Rightarrow \neg likes(X,Y)$$

The only vegetarian butchers are women.

$$\forall X\ vegetarian(X) \wedge butcher(X) \Rightarrow female(X)$$

No man likes a woman who is a vegetarian.

$$\neg\ \exists X\ \exists Y\ male(X) \wedge female(Y) \wedge vegetarian(Y) \wedge likes(X,Y)$$

$$\forall X\ \neg \exists Y\ ...$$

(says the same thing)

No woman likes a man who does not like all vegetarians.

$\neg \exists X \exists Y \forall Z$ female(X) $\wedge$ male(Y) $\wedge$ vegetarian(Z) $\wedge$ $\neg$ likes(Y,Z) $\wedge$ likes(X,Y)

Given two sentences phi and chi:
(ϕ = phi, χ = chi)

$\forall X\ \phi$ (usually ϕ has "$\Rightarrow$" in it)

$\exists X\ \chi$ (usually χ has all "$\wedge$"'s in it)

We can push the negation inward:
$\forall X\ \neg \exists Y\ \forall Z\ \phi$

means the same as:
$\forall X\ \forall Y\ \neg \forall Z\ \phi$

which is the same as:
$\forall X\ \forall Y\ \exists Z\ \neg \phi$

Predicate calculus allows for:

Rule-based deduction

Forward and backward chaining

Clausal form:

Allows for resolution, but we must convert predicate calculus sentences to clausal form.

No man is both a vegetarian and a butcher.

$\neg(\exists X \text{ male}(X) \wedge \text{butcher}(X) \wedge \text{vegetarian}(X))$

could also be written as:

$\neg \exists X \text{ is\_a\_male}(X) \wedge \text{is\_a\_butcher}(X) \wedge \text{is\_a\_veg}(X)$

Rule 1: if P1 then C1

Rule 2: if P2 then C1

Knowledge: P1 = true, CF = 0.5
P2 = true, CF = 0.4

Rule 1 fires (modus ponens, rule-based deduction):

C1 = true, CF = $(0.5 \times 1) = 0.5$

Rule 2 fires:

C1 = true, CF = $(0.4 \times 1) = 0.4$

Combine certainty factors:

$$CF = 0.5 + (1-0.5) \times 0.4 = 0.7$$

(See pages 329-331 in textbook)

Combining same-sign certainty factors:

$$CF(R1) + CF(R2) - (CF(R1) \times CF(R2))$$

$$0.5 + 0.4 - (0.5 \times 0.4) = 0.7$$

Combining opposite-sign certainty factors:

$$\frac{CF(R1)-CF(R2)}{1-\min(|CF(R1)|,|CF(R2)|)}$$

Use the above when:
(CF(R1) is + and CF(R2) is -) OR
(CF(R1) is - and CF(R2) is +)

Predicate Calculus

Forward Chaining (sec. 11.2)

Fido goes wherever John goes.

$\forall X\ place(X) \wedge at(john,X) \Rightarrow at(fido,X)$

$P \Rightarrow Q\ =\ \neg P \vee Q$

$\forall X\ \neg(place(X) \vee at(john,X)) \vee at(fido,X)$

John is at the library.

at(john,library)

1. $\neg place(X) \vee \neg at(john,X) \vee at(fido,X)$
2. at(john,library)

Where is Fido?

Resolution of clauses 1 and 2:

$\neg place(library) \vee at(fido,library)$

false *true*

{library/X} < - unifier

Unification + Resolution = Prolog

Automated reasoning; programming in logic.

Resolution is a **complete** inferencing method.

Soundness - if domain only represents true facts represented and uses only sound inferencing methods (i.e. forward/backward chaining) one can only derive true things.

Completeness - anything which is true in any universe of discourse can be derived using a complete method.

Converting Predicate Calculus to Clausal Form

1. $\forall X$ father(X) $\Rightarrow$ parent(X)

2. $\forall X$ $\neg$father(X) $\vee$ parent(X) (I.O.)

3. $\neg$father(X) $\vee$ parent(X) (U.O.)

4. {$\neg$father(X), parent(X)} (O.O.)

yields the resulting clause.

Clauses are also called "well-formed" forms (abbreviated WFF's).

Unification:

color(tweety,yellow)   color(X,Y)
{tweety/X, yellow/Y}

color(tweety,yellow)   color(X,X)
{} can't do it.

color(hat(postman),blue) color(hat(Y),X)
{postman/Y, blue/X}

r(f(X),b)   r(Y,Z)
{f(X)/Y, b/Z}

r(f(Y),X)   r(X,f(b))
{f(b)/X, b/Y} f(Y)/X = F(b)/X

loves(X,Y)   loves(Y,X)
{X/Y} = {Y/X}

r(f(Y),Y,X) r(X,f(a),f(V))

1. f(a)/Y

2. f(Y)/X

3. f(V)/X

4. f(V)/f(Y) (2,3)

5. f(a)/V, f(a)/Y

6. f(f(a))/X

Resolution - working examples (propositions)

<u>Example 1:</u>

1. $\{P,Q\} \equiv P \vee Q \equiv \neg P \Rightarrow Q$

2. $\{\neg P,R\} \equiv \neg P \vee R \equiv P \Rightarrow R$

Resolving 1 and 2 yields:

3. $\{Q,R\} \equiv Q \vee R \equiv \neg Q \Rightarrow R$

so we've derived a new rule.

<u>Example 2:</u>

$\{P,Q\}$

$\{\neg P,Q\}$

Conclude $\{Q\}$

<u>Example 3:</u>

$\{\neg P,Q\} \equiv P \Rightarrow Q$

$\{P\} \equiv P$

Conclude $\{Q\}$

Long-named predicates are easy to create:
$\forall X$ not_all_those_who_wander_are_lost(X)

but are not useful in resolution.
Use simple predicates:

likes(X,Y)
dislikes(X,Y) = ¬ likes(X,Y)

easy(X)
hard(X) = ¬ easy(X)

Don't introduce new predicates when negating an old one will do.

$\forall X\ (P(X) \Rightarrow Q(X)) \quad \forall X\ \phi$
$\forall X\ \neg P(X) \vee Q(X)$ (right)

$\neg \forall X\ P(X) \vee Q(X)$
$\exists X\ \neg P(X) \vee Q(X)$ (wrong)
$\exists X\ \neg (P(X) \vee Q(X))$ (is correct)
$\exists X\ \neg P(X) \wedge \neg Q(X)$ (is also correct)

All purple mushrooms are poisonous.

$\forall X\ purple(X) \wedge mushroom(X) \Rightarrow poisonous(X)$

$\forall X\ purple(X) \Rightarrow (mushroom(X) \Rightarrow poisonous(X))$

$\forall X\ mushroom(X) \Rightarrow (purple(X) \Rightarrow poisonous(X))$

A mushroom is poisonous only if it's purple.

$\forall X\ mushroom(X) \wedge poisonous(X) \Rightarrow purple(X)$

$\forall X\ mushroom(X) \Rightarrow (poisonous(X) \Rightarrow purple(X))$

No purple mushrooms are poisonous.

$\neg \exists X\ purple(X) \wedge mushroom(X) \wedge poison(X)$

$\forall X\ \neg (purple(X) \wedge mushroom(X) \wedge poison(X))$

$\forall X\ \neg purple(X) \vee \neg mushroom(X) \Rightarrow \neg poison(X)$

Resolution and unification = Prolog

Resolution proof - resolution refutation
"Proof by contradiction"

Order of Operations

friends(father(david),father(andrew))
george bill

$\neg$,
$\wedge$, $\vee$,
$\Rightarrow$, =

$\forall$X $\forall$Y father(X,Y) $\vee$ mother(X,Y) $\Rightarrow$ parent(X,Y)

father(bill,joe)

Unification = binding variables to values

{bill/X, joe/Y} unifier

By rule-based deduction conclude:
parent(bill,joe)

value/var = binding

Constants: usa bill hillary
loves(hillary,bill)
loves(bill,hillary)

WFF's, given person(X) $\wedge$ person(Y):

1. $\forall X \; \exists Y$ loves(X,Y)
Everybody loves somebody.

2. $\forall Y \; \exists X$ loves(X,Y)
Everybody is loved by somebody.
(e.g. your mother)

3. $\exists X \; \forall Y$ loves(X,Y)
Somebody loves everybody.
(e.g. Christ; Mother Teresa)

4. $\exists Y \; \forall X$ loves(X,Y)
Somebody is loved by everybody.
(e.g. Bob Hope, Lucille Ball)

$\forall$ ≡ for all ≡ universal quantifier
$\exists$ ≡ there exists ≡ existential qualifier

Proof by contradiction

To prove a theory:

Assume $\neg$theory.
Compare $\neg$theory with known truths.

If we find a contradiction, then
theory must be true.

Resolution refutation

$\{ \neg P, Q \} \equiv P \Rightarrow Q$

$\{P\} \equiv P$

$\{Q\}$ by resolution

Resolution (sec. 11.2):

1 .	{P}	Given
2.	{¬P, Q}	Given
3.	{¬Q, R}	Given
~~4.~~	~~{¬R}~~	~~Given~~
5.	{Q}	1,2
6.	{¬P, R}	2,3
~~7.~~	~~{¬Q}~~	~~3,4~~
8.	{R}	1,6
9.	{¬P}	2,7
10.	{R}	3,5
~~11.~~	~~{¬P}~~	~~4,6~~
12.	{}	5,7
13.	{}	1,11
14.	{}	4,8
15.	{}	4,10

If empty clause {} is derived by resolution, a contradiction exists.

True or false questions can be asked by resolution.

1.	{ f(art,jon) }	Given
2.	{ f(bob,kim) }	Given
3.	{ ¬ f(X,Y), p(X,Y) }	Given

Is Art a parent of Jon?

4.	{ ¬ p(art,jon) }	Goal (Γ)
5.	{ p(art,jon) }	1,3 {art/X, jon/Y}
6.	{ p(bob,kim) }	2,3 {bob/X, kim/Y}
7.	{ ¬ f(art,jon) }	3,4 {art/X, jon/Y}
8.	{}	4,5
9.	{}	1,7

Since Γ is false, ¬ Γ is true.

Art is the parent of Jon.

Asking fill-in-the-blank questions

1.	{ f(art,jon) }	Given
2.	{ f(bob,kim) }	Given
3.	{ ¬ f(X,Y), p(X,Y) }	Given

Who is Jon's father?

Introduce answer literal...

4.	{ ¬ f(Z,jon), ans(Z) }	Goal
5.	{ p(art,jon) }	1,3 {art/X, jon/Y}
6.	{ ans(art) }	1,4 {art/Z}

Heads I win; tails you lose.

Use resolution to show that I win.

Facts and rules in predicate calculus form:

1.	$H \Rightarrow W(me)$	
2.	$T \Rightarrow L(you)$	
3.	$\neg H \Rightarrow T$	
4.	$L(you) \Rightarrow W(me)$	

Facts and rules in clausal form:

1.	$\{ \neg H, W(me) \}$	Given
2.	$\{ \neg T, L(you) \}$	Given
3.	$\{ H, T \}$	Given
4.	$\{ \neg L(you), W(me) \}$	Given
5.	$\{ \neg W(me) \}$	Goal

Use resolution to reach goal:

6.	$\{ \neg T, W(me) \}$	2,4
7.	$\{ T, W(me) \}$	1,3
8.	$\{ W(me) \}$	6,7
9.	$\{\}$	5,8

If a course has a final, no students are happy.

If a course is easy, some students are happy.

Use resolution to show:
If a course has a final, the course is not easy.

C = course, S = student, t = taking,
h = happy, f = final, e = easy

1. $\forall C\ \forall S\ f(C) \land t(S,C) \Rightarrow \neg h(S)$
2. $\forall C\ e(C) \Rightarrow \exists S\ t(S,C) \land h(S)$
3. $\forall C\ f(C) \Rightarrow \neg e(C)$

1.	{¬f(C), ¬t(S,C), ¬h(S)}	Given
2.	{ ¬e(C), t(g(C),C) }	Given
3.	{ ¬e(C2), h(g(C2)) }	Given
4.	{ f(cs488) }	Goal
5.	{ e(cs488) }	Goal
6.	{ t(g(cs488),cs488) }	2,5
7.	{ h(g(cs488)) }	3,5
8.	{ ¬t(S,cs488), ¬h(S) }	1,4
9.	{ ¬h(g(cs488)) }	6,8 {S/g(cs488)}
10.	{ }	7,9
11.	{¬f(C),¬h(g(C)),¬e(C) }	1,2

Translation to Clauses
Statement 2:

$$\forall C\; e(C) \Rightarrow \exists S\; t(S,C) \wedge h(S)$$

$$\forall C\; \neg e(C) \vee (\exists S\; t(S,C) \wedge h(S)\; \}$$

$$\neg e(C) \vee (t(g(C),C)) \wedge h(g(C)))$$

$$(\neg e(C) \vee t(g(C),C)) \wedge (\neg e(C) \vee h(g(C)))$$

$$\{\; \neg e(C),\; t(g(C),C)\; \}$$
$$\{\; \neg e(C),\; h(g(C))\; \}$$

$$\{\; \neg e(C1),\; t(g(C1),C1)\; \}$$
$$\{\; \neg e(C2),\; h(g(C2))\; \}$$

Translation to Clauses
Statement 3:

$$\neg \forall C\ f(C)\ \ \neg e(C)$$

$$\neg \forall C\ \ \neg f(C) \vee \neg e(C)$$

$$\exists C\ \ \neg(\neg f(C) \vee \neg e(C))$$

$$\exists C\ f(C) \vee e(C)$$

$$f(cs488) \vee e(cs488)$$

{ f(cs488) }
{ e(cs488) }

Murder Mystery

V = Victor, the victim of murder.
3 suspects: A = Abbott, B = Babbitt,
C = Cabot

Abbott says that Babbitt was Victor's friend and Cabot hated victor.

Babbitt says that he was out of town the day of the murder. He doesn't know Victor.

Cabot says that he saw Abbott and Babbitt with Victor just before the murder.

One of A, B, C is the murderer.

Can we prove using resolution who the murderer is?

Assume all people, except the murderer, are telling the truth.

i(X) ≡ innocent(X)	V = Victor
f(a,b) ≡ friends(a,b)	A = Abbott
l(a,b) ≡ likes(a,b)	B = Babbitt
t(X) ≡ in_town(X)	C = Cabot
k(a,b) ≡ knows(a,b)	X = anyone
w(a,b) ≡ with(a,b)	Y = anyone else

$i(A) \Rightarrow f(B,V)$	1	$\{ \neg i(A), f(B,V) \}$
$i(A) \Rightarrow \neg l(C,V)$	2	$\{ \neg i(A), \neg l(C,V) \}$
$i(B) \Rightarrow \neg t(B)$	3	$\{ \neg i(B), \neg t(B) \}$
$i(B) \Rightarrow \neg k(B,V)$	4	$\{ \neg i(B), \neg k(B,V) \}$
$i(C) \Rightarrow w(A,V)$	5	$\{ \neg i(C), w(A,V) \}$
$i(C) \Rightarrow w(B,V)$	6	$\{ \neg i(C), w(B,V) \}$
$w(X,V) \Rightarrow t(X)$	7	$\{ \neg w(X,V), t(X) \}$
$f(X,Y) \Rightarrow k(X,Y)$	8	$\{ \neg f(X,Y), k(X,Y) \}$
$l(X,Y) \Rightarrow k(X,Y)$	9	$\{ \neg l(X,Y), k(X,Y) \}$
$i(A) \vee i(B)$	10	$\{ i(A), i(B) \}$
$i(A) \vee i(C)$	11	$\{ i(A), i(C) \}$
$i(B) \vee i(C)$	12	$\{ i(B), i(C \}$

Negate goal	&	Add 'ans' literal:
i(X) ∨ ans(X)	13	{ i(X), ans(X) }

Using resolution:

14.	{ ¬ i(A), k(B,V) }	1,8
15.	{ ¬ i(C), t(B) }	6,7
16.	{ ¬ i(A), ¬ i(B) }	4,14
17.	{ ¬ i(C), ¬ i(B) }	3,15
18.	{ i(C), ¬ i(B) }	11,16
19.	{ ¬ i(B) }	17,18
20.	{ ans(B) }	13,19