

EE479 Digital Control Systems

Assignment # 1

Read Ch. 10.1 –10.5

Submit the following problems:

10.3 (a), (b), (c), (d), (e)

$$(a) \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$$

$$(b) K = [0.4 \quad 0.2]$$

$$(c) K = [0.8 \quad 0.2]$$

$$(d) \text{ for (b) } \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{for (c) } \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(e) Use “placepol “ function

10.4 (a), (b), (c)

$$(a) \dot{x} = -0.05x + u, \quad y = x$$

$$(b) k = 0.05$$

(c) Obtain the closed-loop block diagram representation

10.7 (a), (b), (c)

$$(a) G = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$(b) G_{ec}(s) = \frac{3.2s + 6.4}{s^2 + 10s + 32}$$

$$(c) \text{ Find } 1 + G_{ec}(s)G_p(s) \quad \begin{aligned} C.E. &= s^4 + 12s^3 + 52s^2 + 96s + 64 = 0 \\ &= (s^2 + 4s + 4)(s^2 + 8s + 16) = 0 \end{aligned}$$

10.8 (a), (b), (c)

$$(a) G = 0.25$$

$$\hat{x} = -0.3\hat{x} + u + 0.25y, \quad u = -0.05\hat{x}$$

$$(b) G_{ec}(s) = \frac{0.0125}{s + 0.35}$$

$$(c) \text{ Find } 1 + G_{ec}(s)G_p(s) \quad \begin{aligned} C.E. &= s^2 + 0.4s + 0.03 = 0 \\ &= (s + 0.1)(s + 0.3) = 0 \end{aligned}$$

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Assignment # 2

Read Ch. 11.1 –11.6

Submit the following problems:

11.2 (a), (b), (c)

(a) $e(t) = 2e^{-2.107t}$

(b) Use the real-transformation time delay $z^{-1}E(z) = Z[x(kT - T)]$

$$e(t) = 2e^{-2.107(t-0.05)}u(t-0.05)$$

(c) Verify by finding the z-transform of each function

11.5 (a), (b), (c), (d), (e), (f)

(a) Use long division $E(z) = 3z^{-2} + 7.2z^{-3} - \dots$

(b) $e(k) = 60 + 15(0.5)^k - 75(0.9)^k$

(c) Use `dimpulse(num, den, k)`

(d) Verify, (yes)

(e) Final value theorem give the correct answer since $E(z)$ has a simple pole at $z=1$.

(f) Find $e(\infty)$ from (b) and the final value theorem, $e(\infty) = 60$

11.6 (a), (d), (e)

(a) Power series $E(z) = z^{-1} + 1.8z^{-2} + 2.44z^{-3} + \dots$

$$e(0) = 0, \quad e(1) = 1, \quad e(2) = 1.8, \quad e(3) = 2.44, \quad e(4) = 2.952$$

Partial fraction expansion $e(k) = 5 - 5(0.8)^k$, Check

(d) Power series $E(z) = z^{-3} + 1.8z^{-4} + 2.44z^{-5} + \dots$

$$e(0) = 0, \quad e(1) = 0, \quad e(2) = 0, \quad e(3) = 1, \quad e(4) = 1.8, \quad e(5) = 2.44$$

Partial fraction expansion $e(k) = 5 - 7.8125(0.8)^k$ for $k \geq 2$, Check

(e) In MATLAB, use `xk=impulse(num, den,k)` for $k=5$ to verify the power series

also use the command `[r, p, k] = residue(num1, den1)` to verify PFE of $\frac{E(z)}{z}$

11.8 (a), (b), (c)

(a) $x(2) = -2, \quad x(3) = 0, \quad x(4) = 2, \quad x(5) = -2, \quad x(6) = 0, \quad x(7) = 2$

(b) $x(k) = 2.309 \sin(k120)$, check for $x(k)$

(c) Write a MATLAB program and verify

11.9 (a), (b), (c), (d), (e)

(a) $x(0) = 1, \quad x(1) = 4, \quad x(2) = 11, \quad x(3) = 26, \quad x(4) = 57$

(b) Modify MATLAB program in Example 11.6 and verify the results in (a)

(c) $X(z) = \frac{z^3}{(z-1)^2(z-2)}$

(d) $X(z) = 1 + 4z^{-1} + 11z^{-2} + 26z^{-3} + 57z^{-4} + \dots$

(e) $x(k) = -3 - k + 4(2)^k$ check and verify the results in (a)

11.11 (a), (b), (c), (d)

(a) $e(k) = 0.1(1.2)^k$, final value unbounded

(b) $e(k) = 0.1(0.8)^k$, $e(\infty) = 0$

(c) $e(k) = 0.333[1 - (0.7)^k]$, $e(\infty) = 0.333$, apply the final value theorem and verify

(d) $e(k)$ is sinusoidal (find the inverse z-transform) $e(\infty)$ does not exist.

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Assignment # 3

Read Ch. 11.6 –11.9

Submit the following problems:

11.15 (a), (b)

- (a) Write the difference equations for simulation diagram in Figure 11.15 (a) and Figure 11.15(b)
 (b) Obtain the transfer function for each simulation diagram

$$\begin{aligned} \text{Figure 11.15 (a)} \quad \frac{M(z)}{E(z)} &= \frac{b_2 z^2 + b_1 z + b_0}{z^2 + a_1 z + a_0} & \Rightarrow \quad \alpha_i = a_i, \quad \beta_i = b_i \\ \text{Figure 11.15 (b)} \quad \frac{M(z)}{E(z)} &= \frac{\beta_2 z^2 + \beta_1 z + \beta_0}{z^2 + \alpha_1 z + \alpha_0} \end{aligned}$$

11.16 (b)

- (b) $m(k) = 1.8m(k-1) - 0.9m(k-2) + 0.333e(k) - 0.556e(k-1) + 0.3e(k-2)$

11.19 (a), (b), (c), (d), (e), (f)

- (a) Draw the simulation diagram

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.6 & 1.2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$(b) \quad y(k) = \begin{bmatrix} 0.03 & 0.035 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

- (c) Use $\frac{Y(z)}{U(z)} = C[zI - A]^{-1} B$, and show $\frac{Y(z)}{U(z)} = \frac{0.035z + 0.03}{z^2 - 1.2z + 0.6}$

- (d) Obtain directly from the difference equation and verify the result in (c)
 (e) Apply Mason's rule and verify the result in (c)
 (f) Use the function `[Gnum, Gden]= ss2tf(A, B, C, D); G=tf(Gnum, Gden, -1)`

11.21 (b), (f)

- (b) $y(k) = 3.17 - 9.29(0.3)^k + 6.11(0.1)^k$
 (f) `[Gnum, Gden]= ss2tf(A, B, C, D); G=tf(Gnum, Gden, -1)`

11.23 (a), (b), (c)

$$(a) \quad \phi(k) = \begin{bmatrix} -\frac{1}{2}k2^k + 2^k & -\frac{1}{2}k2^k \\ \frac{1}{2}k2^k & \frac{1}{2}k2^k + 2^k \end{bmatrix}$$

$$(b) \quad x(1) = \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \quad x(2) = \begin{bmatrix} 8 \\ -12 \end{bmatrix}, \quad x(3) = \begin{bmatrix} 20 \\ -28 \end{bmatrix},$$

$$(c) \quad x(3) = \begin{bmatrix} 20 \\ -28 \end{bmatrix}$$

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Assignment # 4

Read Ch. 12.1 –12.9

Submit the following problems:

12.1 (a), (d)

$$(a) E^*(s) = \frac{e^{Ts}}{e^{Ts} - e^{-3T}}$$

$$(d) E^*(s) = \frac{\frac{3}{2}e^{Ts}}{e^{Ts} - 1} - \frac{\frac{1}{2}e^{Ts}}{e^{Ts} - e^{-2T}}$$

11.11 (a), (b), (c)

$$(a) c(kT) = e^{-kT} = e^{-0.1k}$$

$$(b) c(t) = e^{-t}$$

(c) No effect, the sampler zero-order hold rebuilds a constant signal exactly

12.13 (a), (b)

$$(a) G(s) = \frac{1 - e^{-Ts}}{s} G_p(s), \text{ find } G(z) = \frac{0.1813}{z - 0.8187}$$

$$A(s) = E(s) \frac{1}{s+1} = \frac{1}{s(s+1)}, \text{ find } A(z) = \frac{0.09516z}{(z-1)(z-0.9048)}$$

$$C(z) = A(z)G(z) = \frac{0.01725z}{(z-1)(z-0.9048)(z-0.8187)}$$

$$c(kT) = 1 = 1.904(0.9048)^k + 0.9047(0.8187)^k$$

(b) Plot $c(kT)$, simulate the system with SIMULINK and verify the plot

12.15 (a), (b), (c)

Correction: Change the given difference equation to $m(k) = m(k-1) + 0.5e(k)$

$$(a) \frac{C(z)}{E(z)} = \frac{0.1z}{z^2 - 2z + 1}$$

(b) Draw the simulation diagram

(c)

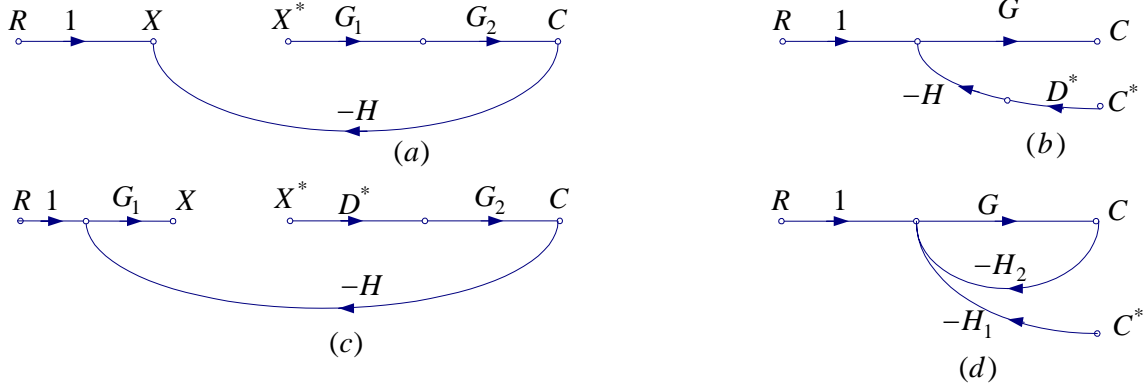
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 0.1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

12.16 (a), (b), (c), (d)

Follow the procedure 1-5 outlined in Chapter 12 (Page 506). Remember if there is a sampler between $G_1(s)$ and $G_2(s)$, in the sampled block diagram the gain is $G_1(z)G_2(z)$.

If there is no sampler between $G_1(s)$ and $G_2(s)$, we have a single transfer function denoted by $\overline{G_1G_2}(z)$.



Determine $\frac{C(z)}{R(z)}$ for each block diagram.

12.17 (a), (b), (c), d

(a) $G(z) = \frac{0.05(z+1)}{z^2 - 2z + 1}$

(b) $T(z) = \frac{0.05k(z+1)}{z^2 - (z - 0.05k)z + (1 + 0.05k)}$

(c)

$D(z) = \frac{2.2z - 1}{z}$

$T(z) = \frac{0.05k(2.2z^2 + 1.2z - 1)}{z^3 - (z - 0.11k)z^2 + (1 + 0.06k)z - 0.05k}$

(d) Define num, den, use $G_s = \text{tf}(\text{num}, \text{den})$, $G_z = \text{c2d}(G_s, 0.1 \text{ 'zho'})$

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Assignment # 5

Read Ch. 13

Submit the following problems:

13.1 (a) through (f) Correction in Figure P13.1 The plant transfer function is

$$G_p(s) = \frac{K}{s+2}$$

(a) $T = 1$ sec $T(z) = \frac{0.432}{z+0.297}$ $c(k) = 0.333[1 - (-0.297)^k]$

(b) $T = 0.1$ sec $T(z) = \frac{0.0906}{z-0.728}$ $c(k) = 0.333[1 - (0.728)^k]$

(c) $T(s) = \frac{1}{s+3}$ $c(t) = 0.333[1 - e^{-3t}]$

(d) Use MATLAB to plot the above responses.

```
Ta=1; ta=0:1:5;
Tb=0.1; tb=0:0.1:5;
tc =0:0.01:5;
cka = 0.333*(1-(-0.297).^ta/Ta);
stairs(ta, cka, 'k'), grid on
hold on                      % Hold current Graph
    Similarly continue for response (b)
    :
    Define the response for (c) and use plot function
    :
```

(e) Comment

(f) Use **c2d**, and **feedback** functions to obtain the sample-data transfer functions and use **ltiview** to obtain the three responses on the same graph.

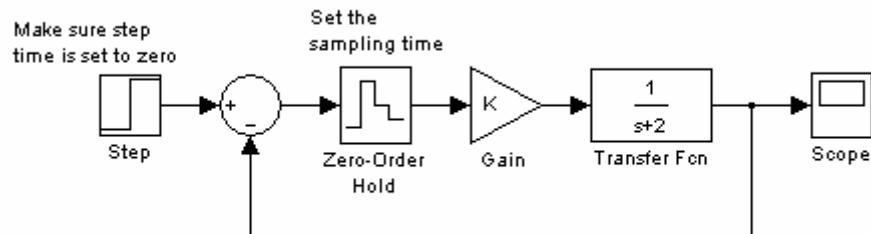
13.2 (a), (b), (c)

$T = 1$ sec $K < 2.63$

(a) $T = 0.1$ sec $K < 20.07$

Continuous $0 < K < \infty$

Include the Simulink diagram and the responses for



For $T = 1$ obtain three responses for $K > 2.63$, $K = 2.63$, $K < 2.63$

For $T = 0.1$ obtain three responses for $K > 20.07$, $K = 20.07$, $K < 20.07$

Comment, is the continuous-time system stable for all $0 < K < \infty$?

(b) For $K = 8$, $T=0.255$ sec

(c) Obtain the Simulink responses for $K = 8$, $T=0.27$, $T =0.255$, $T = 0.2$, and comment.

13.6 (a) Use Bilinear transformation $z = \frac{1+W}{1-W}$ and Routh array instead of Jury test

(a) $0 < K < 5$

13.8 (a), (b), (c), (d), (e), (f)

(a) $G(z) = \frac{0.4261(z+0.8467)}{(z-1)(z-0.6065)}$ Plot the z-plane root locus, Find j-w axis intersect,

breakaway and reentry points.

(b) Assume RL intersect with unit circle is at $0.564+j0.825$, Find $K = 1.1$

(c) $K = 0.0552$ for double roots at $z = 0.791$

(d) Plot root locus using MATLAB and verify (a)

(e) Verify (b)

(f) Verify (c) by plotting the step response in MATLAB

13.13 (a), (b), (c)

Use Bilinear transformation $z = \frac{1+W}{1-W}$ and Routh array instead of Jury test

(a) $0 < K < 1.096$

(b) For $K = 1.096$ find roots of the C.E.

13.16 (a), (b), (c), (d)

(a) $T = 1$ sec, $G(z) = \frac{0.4323}{z-0.1353}$ Type zero, $e_{ss(step)} = 0.667, e_{ss(ramp)} = \infty$

(b) $T = 0.1$ sec, $G(z) = \frac{0.09063}{z-0.8187}$ Type zero, $e_{ss(step)} = 0.667, e_{ss(ramp)} = \infty$

(c) $G(s) = \frac{1}{s+2}$ Type zero, $e_{ss(step)} = 0.667, e_{ss(ramp)} = \infty$

(d) See problem 13.1(e) and comment.